PCCR 2022: Open Problems

Problem 1 (Sebastian Ordyniak):

Consider the following problem:

Simultaneous Directed Feedback Vertex Set	
Input:	Two directed graphs $D_1 = (V, A_1)$ and $D_2 = (V, A_2)$ having the same vertex set V and an integer k.
Parameter:	k
Question:	Is there a set $A \subseteq A_1 \cup A_2$ of size at most k such that $D_1 - A_1$ and $D_2 - A_2$ are both acyclic?

The questions is whether SDFVS is fixed-parameter tractable but has already been resolved by Roohani Sharma, who just sent me an email one day after the open problem session. She and Marcin Pilipczuck showed that the problem is $W[1]$-hard. She considered this question because it has been also stated as an open problem at AGPA 2022.

Problem 2 (Sebastian Ordyniak):

Various Questions concerning computing backdoor depth as defined in [2]:

- Can backdoor depth to finite constraint languages be computed in fpt-time? What is the best possible approximation ratio?
- Can backdoor depth be used for heterogeneous or scattered classes?

  For instance: Is there an fpt-algorithm parameterized by the backdoor depth into the union of 2CNF and HORN? For this it would be sufficient to develop an fpt-approximation for backdoor depth into this class. If this can be done for every combination of finite constraint languages, then this would generalize the corresponding result for normal backdoors [3].

Problem 3 (Andre Schindler):

Various questions concerning the computation of twinwidth:

- Is there a polytime or fpt-approximation for twinwidth? Note that it is known that even deciding whether a graph has twinwidth 4 is NP-hard [1].
- Can we develop efficient heuristics to compute lower bounds and upper bounds for twinwidth?

Problem 4 (Tzvika Geft):

Various questions about the multi-agent path-finding problem on a 2D grid. The question here is: Given a 2D grid graph and n agents each having a starting vertex s_i and end vertex e_i on the grid. Find a plan that moves every agent one edge at a time without collisions such that every agents

moves from its start to its end vertex. There are various optimization functions that can be considers, e.g., minimum total time and minimum maximum distance over all agents.

In an unpublished result Eduard Eiben, Robert Ganian, and Iyad Kanj have shown that the problem for total time is fpt parameterized by the number of agents but NP-hard already for constant total time.

References

[1] Pierre Bergé, Édouard Bonnet, and Hugues Déprés. Deciding twin-width at most 4 is np-complete. *CoRR*, abs/2112.08953, 2021.

[2] Jan Dreier, Sebastian Ordyniak, and Stefan Szeider. CSP beyond tractable constraint languages. In Christine Solnon, editor, *28th International Conference on Principles and Practice of Constraint Programming, CP 2022, July 31 to August 8, 2022, Haifa, Israel*, volume 235 of *LIPIcs*, pages 20:1–20:17. Schloss Dagstuhl - Leibniz-Zentrum für Informatik, 2022.

[3] Robert Ganian, M. S. Ramanujan, and Stefan Szeider. Discovering archipelagos of tractability for constraint satisfaction and counting. In Robert Krauthgamer, editor, *Proceedings of the Twenty-Seventh Annual ACM-SIAM Symposium on Discrete Algorithms, SODA 2016, Arlington, VA, USA, January 10-12, 2016*, pages 1670–1681. SIAM, 2016.