Tractable Abstract Argumentation via Backdoor-Treewidth

Wolfgang Dvořák, Markus Hecher, Matthias König,
André Schidler, Stefan Szeider, and Stefan Woltran

Institute of Logic and Computation, TU Wien, Vienna, Austria

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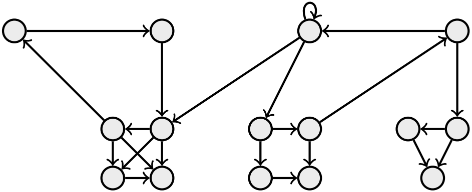
Introduction - Problem

- Argumentation resolves inconsistent information in AI systems
- Standard reasoning tasks computationally expensive
- Exploit structural properties to achieve speedup with **backdoor-treewidth**
- Parameter dominates backdoors and treewidth



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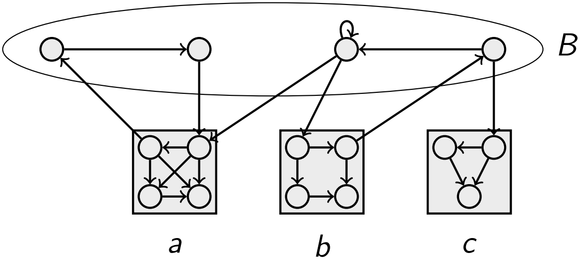

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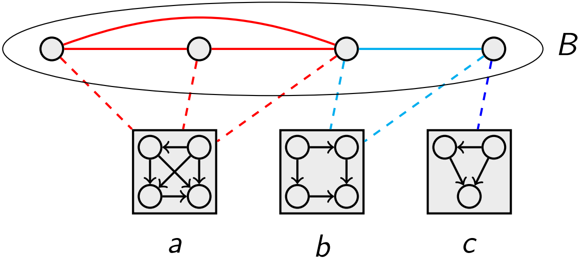
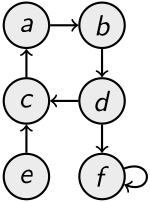


Background - AFs

Definition [Dung, AIJ'95]

An AF $F = (A, R)$ consists of **arguments** A and **attacks** $R \subseteq A \times A$.



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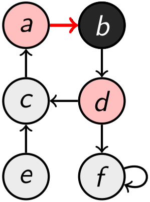
A set $S \subseteq A$...

- ...attacks an $a \in A$ if some $b \in S$ s.t. $(b, a) \in R$



$\{a, d\}$ attacks b

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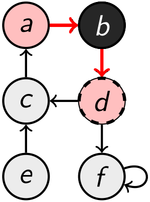
- ...attacks an $a \in A$ if some $b \in S$ s.t. $(b, a) \in R$
- ...defends $a \in A$ if S attacks all attackers of a



$\{a, d\}$ defends d

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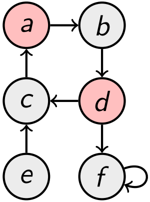
- ...is **conflict-free** it attacks no $a \in S$



$\{a, d\}$ is conflict-free

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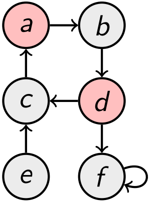
- ...is **admissible** if c.f. and defends all $a \in S$



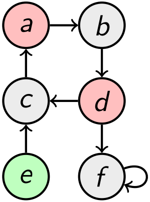
$\{a, d\}$ is admissible

Definition [Dung, AIJ'95]

An AF $F = (A, R)$ consists of **arguments** A and **attacks** $R \subseteq A \times A$.

A set (*extension*) $S \subseteq A$...

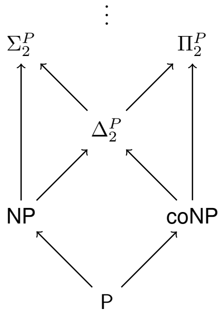
- ...attacks an $a \in A$ if some $b \in S$ s.t. $(b, a) \in R$
- ...defends $a \in A$ if S attacks all attackers of a
- ...is **conflict-free** it attacks no $a \in S$
- ...is **admissible** if c.f. and defends all $a \in S$
- ...is **stable** if c.f. and attacks every $a \in A \setminus E$



$\{a, d\}$ is **not** stable because e is not attacked

Background - Complexity

- We consider credulous/ skeptical reasoning, counting of extensions
- Many problems on first or second level of polynomial hierarchy

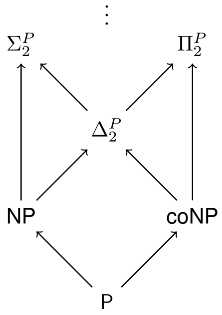


Background - Complexity

- We consider credulous/ skeptical reasoning, counting of extensions
- Many problems on first or second level of polynomial hierarchy
- **Parameterized** algorithms exploit structural properties

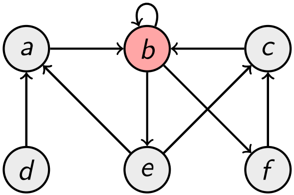
FPT Runtime:

$$O\left(f(p)\cdot n^c\right)$$



Background - Backdoors & Treewidth

- Backdoor-distance allows reasoning in FPT [Dvorak+Ordyniak+Szeider, AIJ'12]
    - Example: $\{b\}$ is a backdoor to acyclicity



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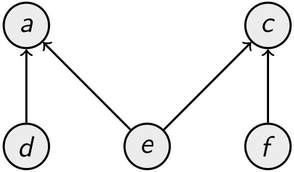
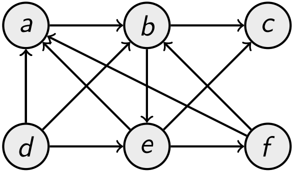


Background - Backdoors & Treewidth

- Backdoor-distance allows reasoning in FPT [Dvorak+Ordyniak+Szeider, AIJ'12]
- Treewidth allows reasoning in FPT [Dvorak+Pichler+Woltran, AIJ'12]
  - Example: high treewidth



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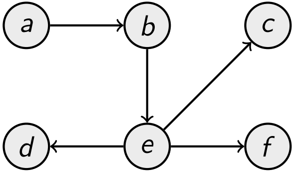
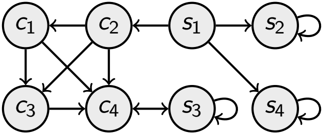
- Backdoor-distance allows reasoning in FPT [Dvorak+Ordyniak+Szeider, AIJ'12]
- Treewidth allows reasoning in FPT [Dvorak+Pichler+Woltran, AIJ'12]
    - Example: small treewidth



Background - Backdoors & Treewidth

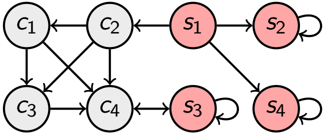
- Backdoor-distance allows reasoning in FPT [Dvorak+Ordyniak+Szeider, AIJ'12]
- Treewidth allows reasoning in FPT [Dvorak+Pichler+Woltran, AIJ'12]
- Backdoor-Treewidth is a combination of both [Ganian+Ramanujan+Szeider, STACS'17]
- Result: Backdoor-Treewidth allows reasoning in FPT with **smaller parameter values**

Backdoor-Treewidth - Example



Backdoor-Treewidth - Example

1. Find a backdoor



Backdoor-Treewidth - Example

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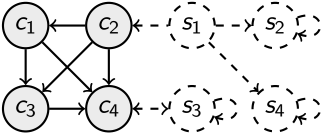
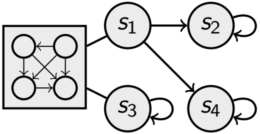


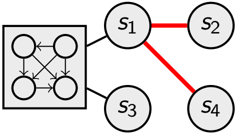
Backdoor-Treewidth - Example

1. Find a backdoor to acyclicity
2. Find the components



Backdoor-Treewidth - Example

1. Find a backdoor to acyclicity
2. Find the components
3. Construct the torso: **attacks**



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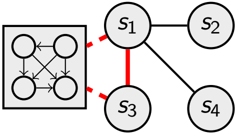
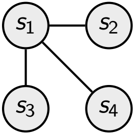


Backdoor-Treewidth - Example

1. Find a backdoor to acyclicity
2. Find the components
3. Construct the torso: **attacks**, **shared components**
4. Reason on simple torso graph

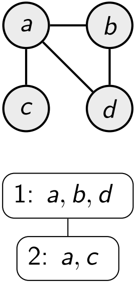


Backdoor-Treewidth

A *tree decomposition (TD)* is a pair $(\mathcal{T}, \mathcal{X})$, where

- $\mathcal{T} = (V_\mathcal{T}, E_\mathcal{T})$ is a tree
- $\mathcal{X} = (X_t)_{t \in V_\mathcal{T}}$ is a set of bags
- $\bigcup_{t \in V_\mathcal{T}} X_t = V$
- for each $v \in V$, the subgraph induced by v in $\mathcal{T}$ is connected
- for each $\{v, w\} \in E$, $\{v, w\} \subseteq X_t$ for some $t \in V_\mathcal{T}$.

Example:



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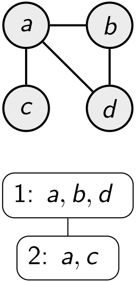
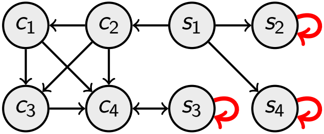
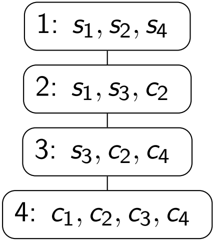


Definition

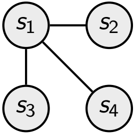
The **backdoor-treewidth** of an AF is the smallest treewidth over **all backdoors** and **all torso-decompositions**.

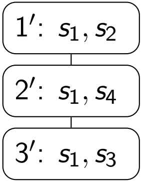
Backdoor-Treewidth - Dominating Parameter





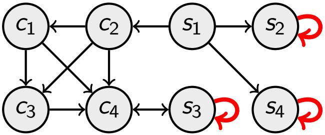
Backdoor: 3
Treewidth: 3





Treewidth: 1

Backdoor-Treewidth - Dominating Parameter



1: s_1, s_2, s_4

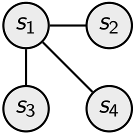
2: s_1, s_3, c_2

3: s_3, c_2, c_4

4: c_1, c_2, c_3, c_4

Backdoor: 3

Treewidth: 3



$1'$: s_1, s_2

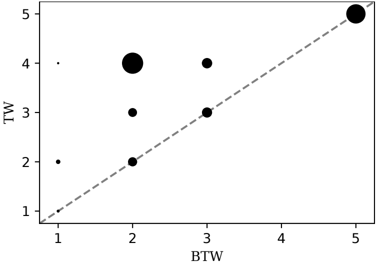
$2'$: s_1, s_4

$3'$: s_1, s_3

Treewidth: 1

Backdoor-Treewidth - In Practice

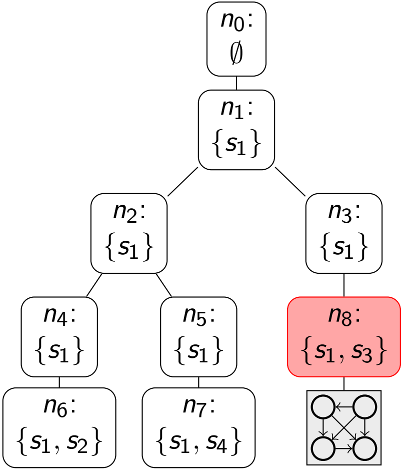
Encoding for exact backdoor-treewidth based on [Samer+Veith, SAT'09].



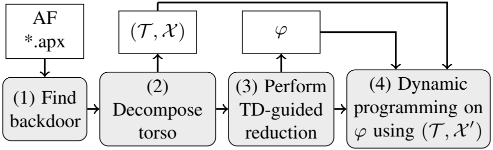
- SAT-formula for each $\texttt{btw}(SF) \leq k$
- $\texttt{btw}(SF) < \texttt{tw}(SF)$ for 611 of the 1134 instances
- $\texttt{btw}(SF) \leq \texttt{tw}(SF)/2$ for 519 out of 611 instances

Exploiting Small Backdoor-Treewidth

- Dynamic Programming on the torso-TD
- Advanced computation in nodes with attached components

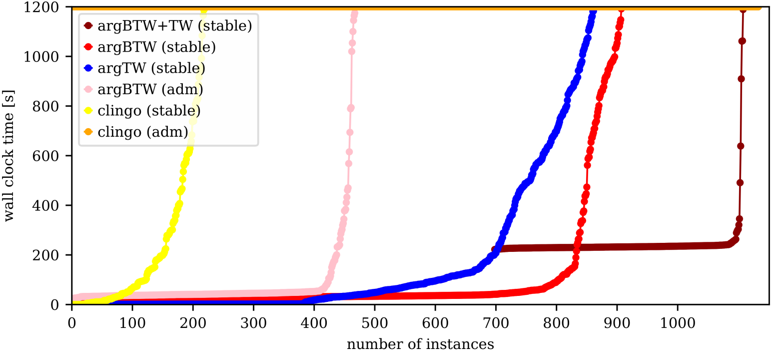


The System argBTW



- `argBTW` implements backdoor-treewidth approach utilizing
  - Heuristics for backdoor and tree-decompositions
  - Tree-decomposition guided reduction to (#)SAT
  - Dynamic Programming on (torso-)decomposition
- Special cases treewidth-only (`argTW`), backdoor-only (`argBW`)
- Supports admissible sets, stable extensions

Performance



Are there instances where `argBTW` outperforms the state-of-the-art?

Summary & Conclusion

- We defined and investigated backdoor-treewidth for abstract argumentation
- We established parameterized tractability by backdoor-treewidth
  - for the fragments acyclicity and even-cycle-freeness
  - for complete and stable extensions
- System `argBTW` outperforms state-of-the-art in certain instances
- Future work: further fragments and semantics, structured approaches