Parameterized algorithmics in Access Control: linking theory and practice

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3rd PCCR, 1st Aug 2022, Haifa, Israel

Outline

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1st PCCR Brno, 2010 (FPT theory: permutation CSPs above average)

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IEEE Symp. ai4i/HCC/GC/Trans-AI 2019 Sep, CA, USA

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(basic) WSP instance: a set $U = \{u_1, u_2, u_3, u_4\}$ of users and a set S of six steps as follows.

s_1 create purchase order	s_2 approve purchase order
s_3 sign goods received note	s_4 countersign goods received note
s_5 create payment	s_6 approve payment

Workflow Satisfiability Problem (WSP), cont'd

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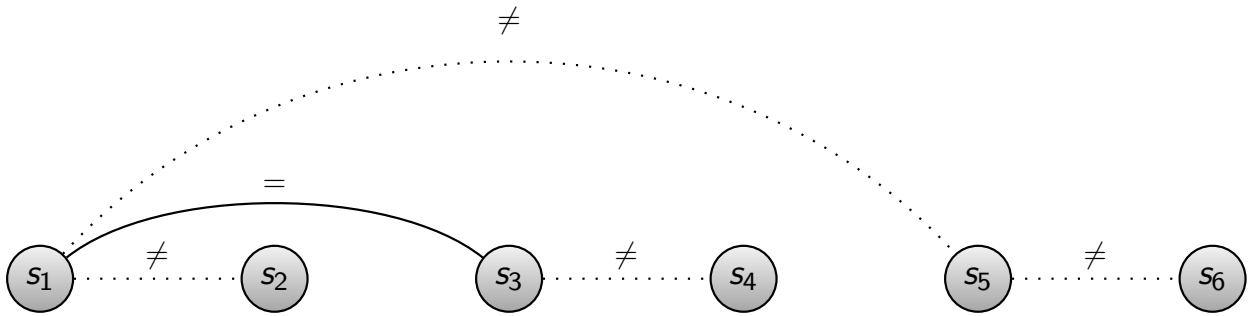
Constraints and authorizations:



Authorized Users					
u_1 u_2 u_3	u_2 u_3	u_3 u_4	u_1 u_3	u_1 u_3 u_4	u_1 u_4

Is there a solution?

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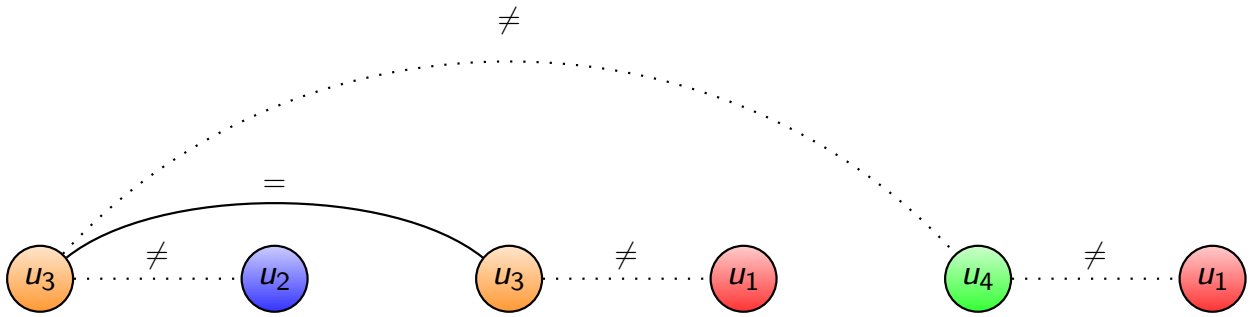

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u_3				u_4	

Is there a solution? Yes.

Definition of basic WSP

- Instance $W = (S, U, C \cup A)$, where S and U are sets of steps and users, C is a family of non-unary constraints with variables S and domain U. Unary constraints (authorizations): $A(s_i) = U'_i \subseteq U$.

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- basic WSP = CSP, where CSP variables = WSP steps, CSP values = WSP users. We'll talk about WSP.

Less Basic WSP

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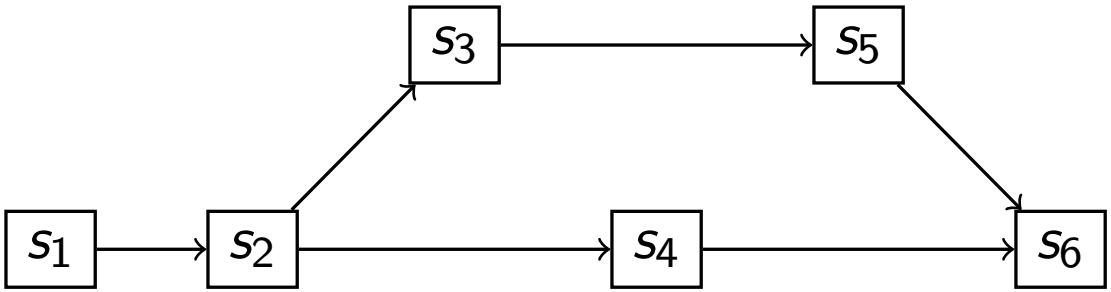

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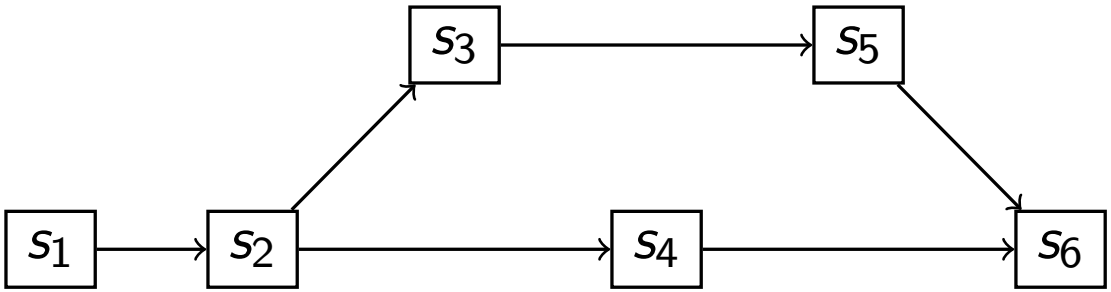

The rest of the talk: Basic WSP. Less Basic WSPs can often be reduced to Basic WSP.

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- Improved and generalized by Crampton, GG and Yeo (ACM Trans. Inf. Syst. Secur., 2013) to regular (undefined here) non-unary constraints with $O^*(2^k)$-algorithm. (Incl.Excl.)

Outline

User-Independent Constraints

- Cohen, Crampton, Gagarin, GG and Jones (JAIR, 2014): constraint c is user-independent (UI) if for every plan $\pi : S \to U$ satisfying c and permutation θ of U, plan $\pi' : S \to U$ also satisfies c, where for each $s \in S$, $\pi'(s) = \theta(\pi(s))$.

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- Counting constraints $(r, Q, \leq)$ and $(r, Q, \geq)$ are UI. $(r, Q, \leq)$ is satisfied by π if $|\pi(Q)| \leq r$ (confidentiality) and $(r, Q, \geq)$ is satisfied by π if $|\pi(Q)| \geq r$ (diversity). Example: AllDiff(Q)=$(|Q|, Q, \geq)$.

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- All the non-unary constraints defined in the 2004 American National Standards Institute Role Based Access Control standard are UI.

Results for Non-unary User-Independent Constraints (Unary Constraints are Arbitrary)

Cohen, Crampton, Gagarin, GG and Jones (JAIR 2014):

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- BFS-like user-iterative algorithm with exponential space.

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- Karapetyan, Gagarin and GG (FAW 2015): a backtracking $O^*(2^{k \log k})$-time poly-space algorithm for k-WSP with UI non-unary constraints, much faster in practice. Further improved by Karapetyan, Parks, GG and Gagarin (JAIR 2019).

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- Gutin and Wahlström (IPL 2016): k-WSP with UI non-unary constraints cannot be solved in time $O^*(c^{k \log k})$ for any $c < 2$ unless Strong ETH fails.

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Basic Algorithm

- Generate all (unordered) partitions (called patterns) of S into non-empty subsets. [Runtime $O^*(B_k) = O^*(2^{k \log k})$, where B_k is k'th Bell number, $B_k \leq k!$.]
- Remove all of the patterns, which do not satisfy at least one non-unary constraint. [Runtime $O^*(B_k)$ provided that checking each non-unary constraint satisfaction takes $O^*(1)$ time.]

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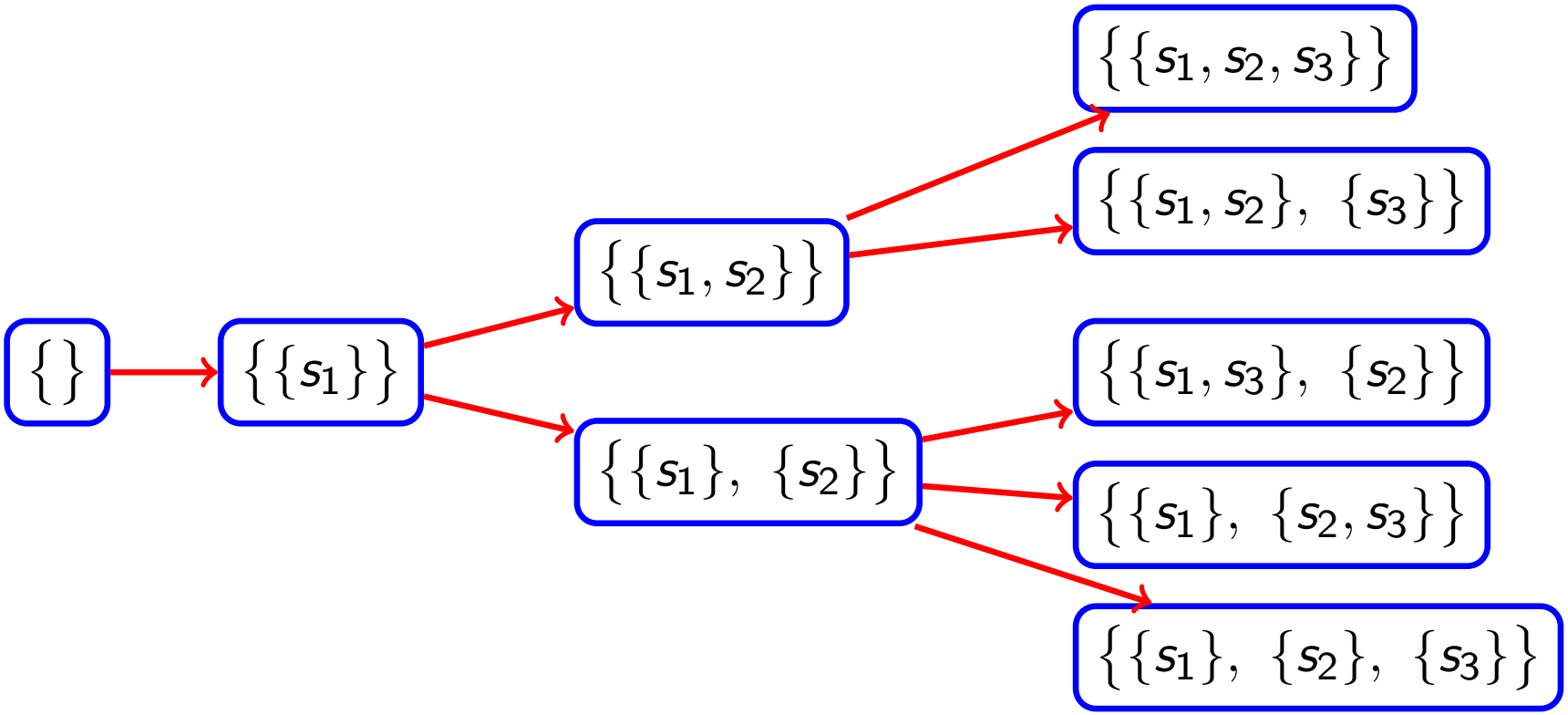
- For each pattern $S_1 \uplus \cdots \uplus S_t = S$ construct a bipartite graph B with bipartition $(\{S_1, \ldots, S_t\}, U)$ s.t. $S_i u_j \in E(B)$ if u_j is authorised for all steps in S_i. If B has a matching M of size t, then return π with $\pi(S_i) = u_j$ for every $S_i u_j \in M$. [Runtime $O^*(B_k)$.]

Generating Patterns for Backtracking



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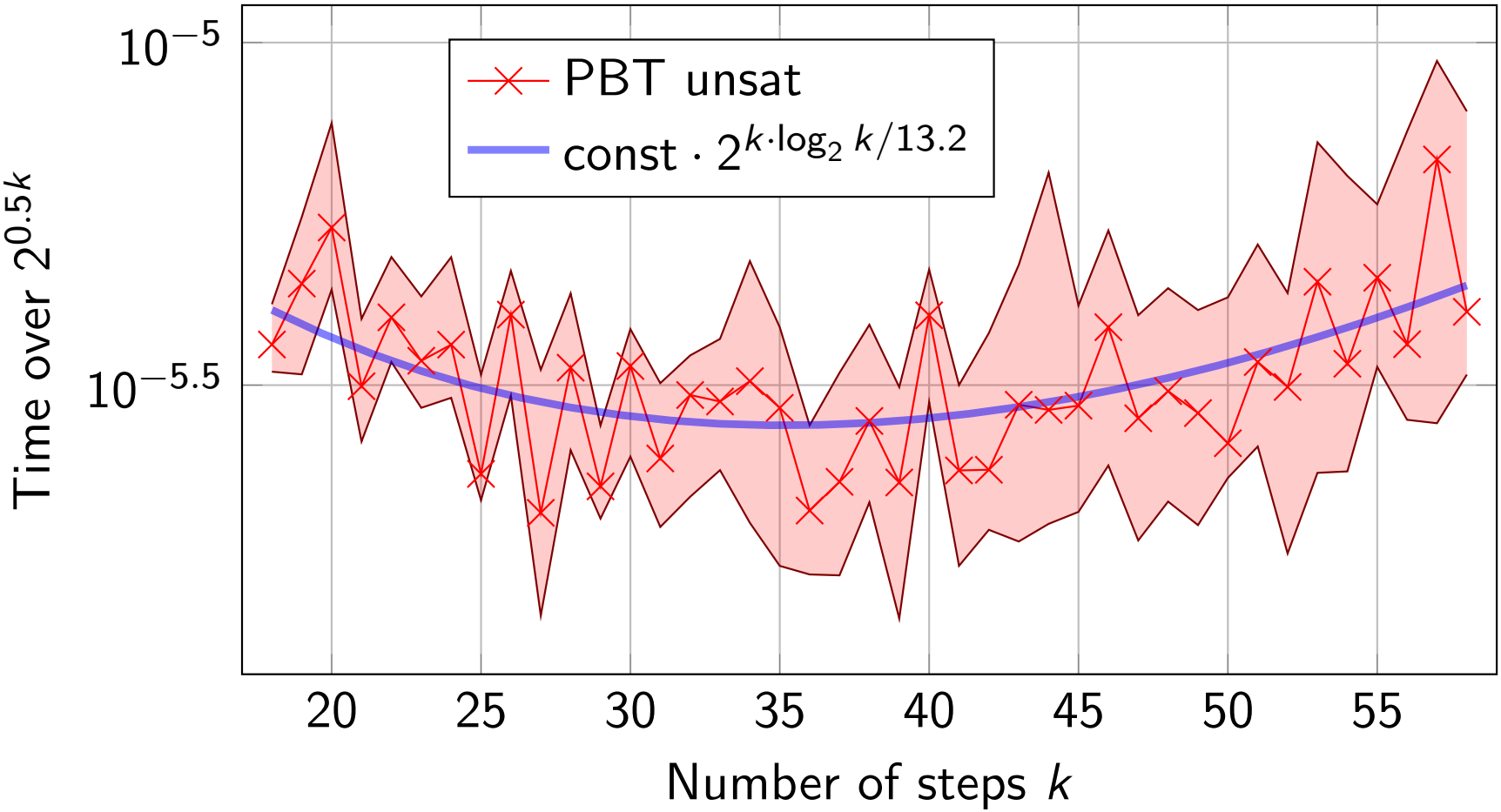
  - k at-most-3-out-of-5 $c = (T, \leq)$, where $T \subseteq S$ of size 5. A plan π satisfies c if $|\pi(T)| \leq 3$.
  - e non-equals $c = (s_i, s_j, \neq)$: plan π satisfies c if $\pi(s_i) \neq \pi(s_j)$. e is chosen s.t. $\mathbb{P}[(S, U, C)$ being satisfiable$]$ is approx 50%. They are hardest instances.


PBT unsat
const · $2^{k \cdot \log_2 k/13.2}$
Time over $2^{0.5k}$
10^{-5}
$10^{-5.5}$
20
25
30
35
40
45
50
55
Number of steps k

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CP-SAT CP-SAT from OR-tools (Google) using 'ordinary' CSP formulation of WSP

Ordinary Pseudo-Boolean formulations of WSP

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- $x_{s,u} = 1$ iff u is assigned to s.
- 'Ordinary' formulation (only not-equals):

$$\begin{aligned} \textstyle\sum_{u \in U} x_{s,u} = 1 \quad & \forall s \in S, \\ x_{s,u} = 0 \quad & \forall s \in S \text{ and } \forall u \in U \setminus A(s), \\ x_{s_1,u} + x_{s_2,u} \leq 1 \quad & \forall \text{ not-equals with scope } \{s_1, s_2\} \text{ and } \forall u \in U. \end{aligned}$$

FPT Formulation

'FPT' formulation uses $x_{s,u}$'s and 'FPT variables' $M_{s,s'}$. $M_{s,s'} = 1$ if s, s' are assigned the same user and $M_{s,s'} = 0$, otherwise. Much longer formulation.

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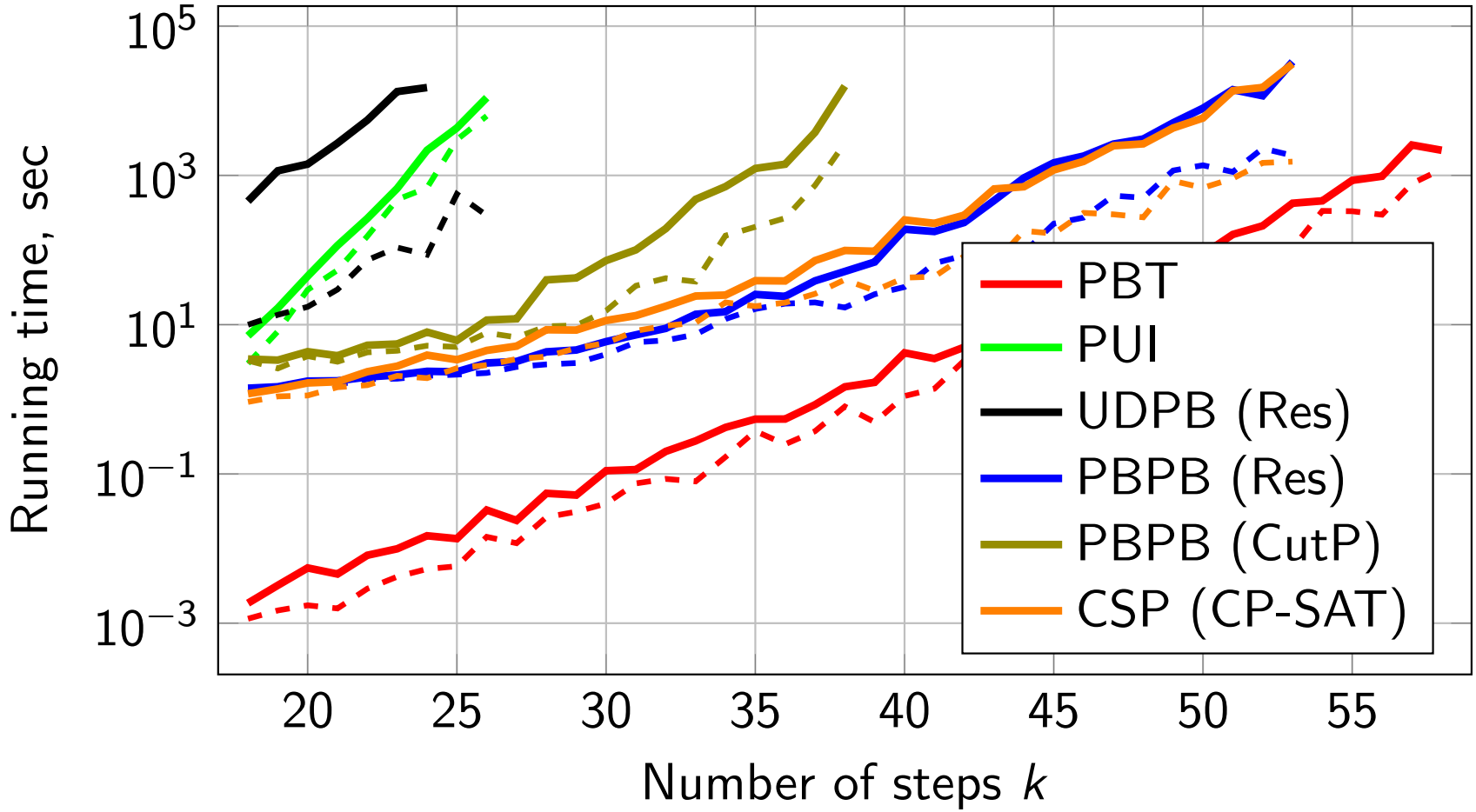
Theorem

Every non-binary UI constraint can be encoded using $M_{s,s'}$'s.


Running time, sec
10^5
10^3
10^1
10^{-1}
10^{-3}
20
25
30
35
40
45
50
55
Number of steps k
PBT
PUI
UDPB (Res)
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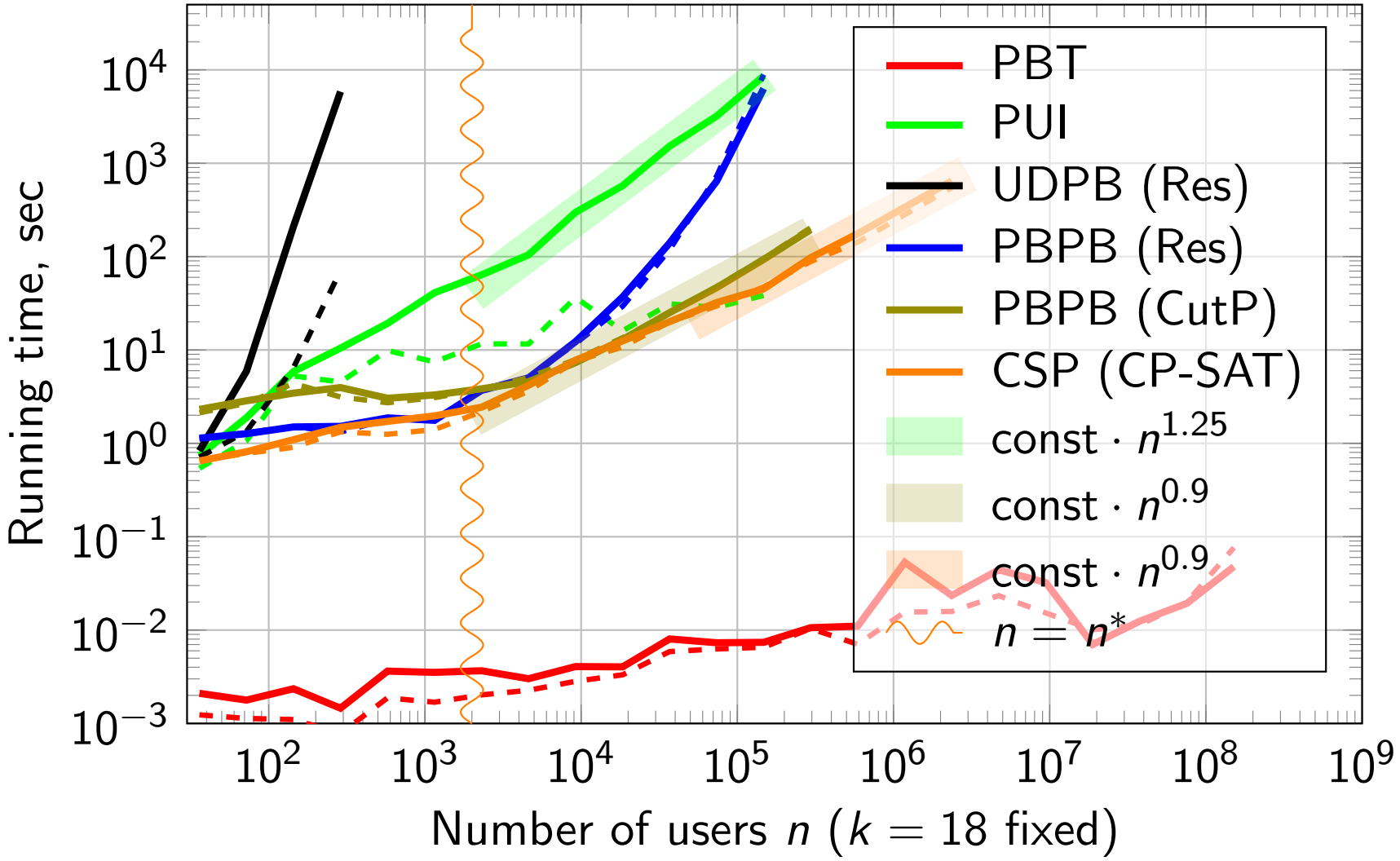
Running time, sec
10^4
10^3
10^2
10^1
10^0
10^{-1}
10^{-2}
10^{-3}
10^2
10^3
10^4
10^5
10^6
10^7
10^8
10^9
Number of users n ($k = 18$ fixed)
PBT
PUI
UDPB (Res)
PBPB (Res)
PBPB (CutP)
CSP (CP-SAT)
const $\cdot\, n^{1.25}$
const $\cdot\, n^{0.9}$
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$n = n^*$

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- Likely explanation: CP-SAT formulation is reduced to a SAT formulation. 'FPT variables' appear in the SAT formulation.

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- A constraint c is UI (for Valued WSP) if $w_c(\pi) = w_c(\pi')$, where π and π' are equivalent, i.e. the same up to permutation of users.
- If all non-unary constraints are UI, $O^*(2^{k \log k})$-algorithm.

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- C_1 unary constraints, $w_1(\pi) = \sum_{c \in C_1} w_c(\pi)$, C_2 non-unary constraints, $w_2(\pi) = \sum_{c \in C_2} w_c(\pi)$.

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- Find Pareto front (Pf) of $\text{argmin}_\pi(w_1(\pi), w_2(\pi))$. $\pi \in$ Pf iff $\nexists\, \pi'$ with $w_1(\pi') + w_2(\pi') < w_1(\pi) + w_2(\pi)$, $w_1(\pi') \leq w_1(\pi)$, and $w_2(\pi') \leq w_2(\pi)$.

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- Find Pareto front (Pf) of $\text{argmin}_\pi(w_1(\pi), w_2(\pi))$. $\pi \in$ Pf iff $\nexists\, \pi'$ with $w_1(\pi') + w_2(\pi') < w_1(\pi) + w_2(\pi)$, $w_1(\pi') \leq w_1(\pi)$, and $w_2(\pi') \leq w_2(\pi)$.
- If all non-unary constraints are UI, $O^*(2^{k \log k})$-algorithm.

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- Introduced by Lu, Vaidya and Atluri (Int. Conf. Data Eng., 2008). Studied by many incl. Fomin, Golovach and Panolan (Data Mining Know. Disc., 2020) who used PC.

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- So, Noise Role Mining should be generalized.

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For Noise Role Mining, set $\mathbf{f}_{ij} = \top$ for all i, j. For Security-preserving Role Mining, set $\mathbf{f}_{ij} = \bot$ iff $\mathbf{UPA}_{ij} = 0$.

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- **Theorem** (Crampton et al., SACMAT 2022) Noise/Availability-preserving/Security-preserving Cover Role Mining parameterized by $r + \mathrm{cov}_U + \mathrm{cov}_P$ admits an FPT algorithm.

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- Interesting outcome for CP-SAT solver: FPT-like runtime for all three types of instances.
- Usually, it's much faster to solve SECURITY instances than NOISE and AVAILABILITY ones.

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- If an NP-hard problem is FPT, off-the-shelf solvers may be made more efficient if some variables used in the constraints correspond to the parameter(s).
- Some solvers may take advantage of NP-hard problems being FPT without any help from the users.